

Prob. Let A,B,C be any three sets, then prove that-

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

Solution:

$$(x,y) \in A \times (B \cap C)$$

$$x \in A \text{ and } (y \in (B \cap C))$$

$$x \in A \text{ and } (y \in B \text{ and } y \in C)$$

$$(x \in A \text{ and } y \in B) \text{ and } (x \in A \text{ and } y \in C)$$

$$(x,y) \in (A \times B) \text{ and } (x,y) \in (A \times C) // \text{ by Cartesian Product.}$$

$$(x,y) \in (A \times B) \cap (A \times C)$$

Prob. Prove that-

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Solution:

$$\text{Let } x \in A \cap (B \cup C).$$

$$\text{Then } x \in A \text{ and } x \in (B \cup C).$$

$$(x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C).$$

$$x \in (A \cap B) \text{ or } x \in (A \cap C).$$

$$x \in (A \cap B) \cup (A \cap C).$$

Prob. If A, B, C, D are any four sets then prove that -

$$(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$$

Solution:

Consider (x,y)

$$(x,y) \in (A \cap B) \times (C \cap D)$$

$$x \in (A \cap B) \wedge y \in (C \cap D)$$

$$(x \in A \text{ and } x \in B) \wedge (y \in C \text{ and } y \in D)$$

$$(x \in A \wedge y \in C) \text{ and } (x \in B \wedge y \in D)$$

$$(x,y) \in (A \cap C) \text{ and } (x,y) \in (B \cap D)$$

$$(x,y) \in ((A \cap C) \text{ and } (B \cap D))$$

$$(x,y) \in ((A \times C) \cap (B \times D))$$

$$(A \times C) \cap (B \times D)$$

Prob. Show that-

$$(P \cap Q) \times (R \cap S) = (P \times R) \cap (Q \times S)$$

For some arbitrary sets P, Q, R and S

Solution:

Consider (x,y)

$$(x,y) \in (P \cap Q) \times (R \cap S)$$

$$x \in (P \cap Q) \wedge y \in (R \cap S)$$

$$(x \in P \text{ and } x \in Q) \wedge (y \in R \text{ and } y \in S)$$

$$(x \in P \wedge y \in R) \text{ and } (x \in Q \wedge y \in S)$$

$$(x,y) \in (P \cap R) \text{ and } (x,y) \in (Q \cap S)$$

$$(x,y) \in ((P \cap R) \text{ and } (Q \cap S))$$

$$(x,y) \in ((P \times R) \cap (Q \times S))$$

$(PXR) \cap (QXS)$

Related Posts:

1. SET
2. Mathematical induction
3. Net 34
4. prove that- $AX(B \cap C) = (AXB) \cap (AXC)$
5. Prove that- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
6. prove that - $(A \cap B) \cup (C \cap D) = (A \cup C) \cap (B \cup D)$
7. Show that- $(P \cap Q) \cup (R \cap S) = (P \cup R) \cap (Q \cup S)$
8. Binary operations
9. Algebraic structure
10. Group
11. Show that $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$ is group
12. Show that $a*b=b*a$
13. if $a*c = c*a$ and $b*c = c*b$, then $(a*b)*c = c*(a*b)$
14. Undirected Graph and Incident Matrix
15. Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$, $1^3 + 2^3 + 3^3 + \dots + n^3 = [n(n+1)/2]^2$
16. Prove that $G=\{-1,1,i,-i\}$ is a group under multiplication.
17. Hasse diagram for the "less than or equal to" relation on the set $S= \{ 0,1,2,3,4,5\}$