

if $a*c = c*a$ and $b*c = c*b$, then $(a*b)*c = c*(a*b)$

Let $(A, *)$ be a semigroup. Show that for $a, b, c \in A$, if $ac = ca$ and $bc = cb$, then $(ab)c = c(a*b)$.

Solution:

Given, $a*c = c*a$
 $b*c = c*b, \forall a, b, c \in A$

To Show,
 $(a*b)*c = c*(a*b)$

Taking LHS,

$$\begin{aligned}
 (a*b)*c &= a*(b*c) && [\text{Using associative law}] \\
 &= a*(c*b) && [\because b*c = c*b] \\
 &= (a*c)*b && [\text{Using associative law}] \\
 &= (c*a)*b && [\because a*c = c*a] \\
 &= c*(a*b) && [\text{Using associative law}]
 \end{aligned}$$

Hence proved.